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Vibration and Buckling Analysis of Cracked Composite Beam

Ranveer Sahu[1], Dr. Ajay Swarup[2]
M.Tech Scholar, Department of Civil Engineering Sri Satya Sai University of Technology & Medical Science, Sehore, Bhopal [1]
Associate Professor, Department of Civil Engineering Sri Satya Sai University of Technology & Medical Science, Sehore, Bhopal[2]
ranveersahu383@gmail.com[1], ajay0381@yahoo.com[2]

Abstract

Cracks in structural members lead to local changes in their stiffness and consequently their static and dynamic behaviour is altered. The influence of cracks on dynamic characteristics like natural frequencies, modes of vibration of structures has been the subject of many investigations. However, studies related to behaviour of composite cracked structures subject to in-plane loads are scarce in literature. Present work deals with the vibration and buckling analysis of a cantilever beam made from graphite fibre reinforced polyimide with a transverse one-edge non-propagating open crack using the finite element method. The undamaged parts of the beam are modelled by beam finite elements with three nodes and three degrees of freedom at the node. An overall additional flexibility matrix" is added to the flexibility matrix of the corresponding non-cracked composite beam element to obtain the total flexibility matrix, and therefore the stiffness matrix in line with previous studies. The vibration of cracked composite beam is computed using the present formulation and is compared with the previous results. The effects of various parameters like crack location, crack depth, volume fraction of fibres and fibres orientations upon the changes of the natural frequencies of the beam are studied. It is found that, presence of crack in a beam decreases the natural frequency which is more pronounced when the crack is near the fixed support and the crack depth is more. The natural frequency of the cracked beam is found to be maximum at about 45% of volume fraction of fibres and the frequency for any depth of crack increases with the increase of angle of fibres. The static buckling load of a cracked composite beam is found to be decreasing with the presence of a crack and the decrease is more severe with increase in crack depth for any location of the crack. Furthermore, the buckling load of the beam decreased with increase in angle of the fibres and is maximum at 0-degree orientation.

Keywords: *Vibration and Buckling Analysis, Cracked Composite Beam, Graphite fibre reinforced polyimide ,crack location ,crack depth ,volume fraction of fibres etc.*

1. Introduction

1.1 Introduction

Composites as structural material are being used in aerospace, military and civilian applications because of their tailor made properties. The ability of these materials to be designed to suit the specific needs for different structures makes them highly desirable. Improvement in design, materials and manufacturing technology enhance the application of composite structures. The suitability of a particular composite material depends on the nature of applications and needs. The technology has been explored extensively for aerospace and civil engineering applications, which require high strength and stiffness to weight ratio materials. Cracks or other defects in a structural element influence its dynamical behaviour and change its stiffness and damping properties. Consequently, the natural frequencies and mode shapes of the structure contain information about the location and dimensions of the damage. Vibration analysis can be used to detect structural defects such as cracks, of any structure offer an effective, inexpensive and fast means of nondestructive testing. What types of changes occur in the vibration characteristics, how these changes can be detected and how the condition of the structure is interpreted has been the topic of several research studies in the past. The use of composite materials in various construction elements has increased substantially over the past few years.

1.2 Scope of the present Investigation

The main aim of this thesis is to work out a composite beam finite element with a non-propagating one-edge open crack. It has been assumed that the crack changes only the stiffness of the element whereas the mass of the element is unchanged. For theoretical modelling of cracked composite beam dimensions, crack locations, crack depth and material properties is specified. In this work an "overall additional flexibility matrix", instead of the "local additional flexibility matrix" is added to the flexibility matrix of the corresponding non-cracked composite beam element to obtain the total flexibility matrix, and therefore the stiffness matrix in the line with the other researchers. By using the present model the following effects due to the crack of the cantilever composite beam have been analyzed. The influence of the volume fraction of fibers, magnitude, location of the crack, angle of fibers upon the bending natural frequencies of the cantilever cracked composite beam. The effects of above parameters on buckling analysis of cracked composite beam. The present results are compared with previous studies and the new results are obtained in the MATLAB environment.

2. Literature Review

2.1 Introduction

The widespread use of composite structures in aerospace applications has stimulated many researchers to study various aspects of their structural behavior. These materials are particularly widely used in situations where a large strength-to-weight ratio is required. Similarly, to isotropic materials, composite materials are subjected to various types of damage, mostly cracks and delamination. These result in local changes of the stiffness of elements for such materials and consequently their dynamic characteristics are altered. This problem is well understood in case of constructing elements made of isotropic materials, while data concerning the influence of fatigue cracks on the dynamics of composite elements are scarce in the available literature.

2.2 Review on vibration of cracked composite beam

A local flexibility will reduce the stiffness of a structural member, thus reducing its natural frequency. For small crack depths the change (decrease) in natural frequency is proportional to the square of the crack depth ratio.

3. Theory and Formulation

3.1 Introduction

Structures are debilitated by cracks. At the point when the break size increments in course of time, the structure gets to be weaker than its past condition. At last, the structure may breakdown because of a moment split. The fundamental design of the issue examined here is a composite beam of any limit condition with a transverse one-edge nonengendering open split. Be that as it may, a run of the mill broke cantilever composite beam, which has enormous applications in aviation structures and fast turbine hardware, is considered.

The following aspects of the crack greatly influence the dynamic response of the structure.

- The position of a crack in a cracked composite beam
- The depth of crack in a cracked composite beam
- The angle of fibers in a cracked composite beam
- The volume fraction of fibers in a cracked composite beam

3.2 Methodology

The governing equations for the vibration analysis of the composite beam with an open one-edge transverse crack are developed. An additional flexibility matrix is added to the flexibility matrix of the corresponding composite beam element to obtain the total flexibility matrix and therefore the stiffness matrix is obtained by Krawczuk & Ostachowicz (1995).

The assumptions made in the analysis are:

- The analysis is linear. This implies constitutive relations in generalized Hook's law for the materials is linear.
- The Euler–Bernoulli beam model is assumed.
- The damping has not been considered in this study.
- The crack is assumed to be an open crack and have uniform depth "a"

3.3 Computational procedure for a cracked composite beam

A computer program is developed to perform all the necessary computations in MATLAB environment. In the initialization phase, geometry and material parameters are specified. For example, for a Euler– Bernoulli composite beam model with localized crack, material parameters like modulus of elasticity, the modulus of rigidity, the Poisson ratio and the mass density of the composite beam material and geometric parameters like dimensions of the composite beam, also the specifications of the damage like size of the crack, location of the crack and extent of crack are supplied as input data to the computer program.

A. Governing Equation

The differential equation of the bending of a beam with a mid-plane symmetry ($B_{ij} = 0$) so that there is no bendingstretching coupling and no transverse shear deformation ($\varepsilon = 0$) is given by;

$$IS_{11}\frac{d^4\omega}{dx^4} = q\ x$$

For a composite beam in which different lamina have differing mass densities, then in the above equation use, for a beam of rectangular cross-section,

$$\rho A = \rho bh = {}_{k=1}^{N}\rho b\ \ h_k - h_{k-1}$$

Thus, the natural vibration equation of a mid-plane symmetrical composite beam is given by;

$$IS_{11}\frac{d^4\omega(x,t)}{dx^4} + \rho A\frac{\partial^2\omega(x,t)}{\partial x^2} = 0$$

The natural frequency in radians/unit time are given as

$$\omega_n = \alpha^2 \left(\frac{IS_{11}}{\rho AL^4}\right)^{1\ 2}$$

B. Mathematical Model

The model chosen is a cantilever composite beam of uniform cross-section A, having an open-edge transverse crack of depth „a" at position „***l*₁**'. The width, length and height of the beam are B, L and H, respectively in Figure. The angle between the fibers and the axis of the beam is α".

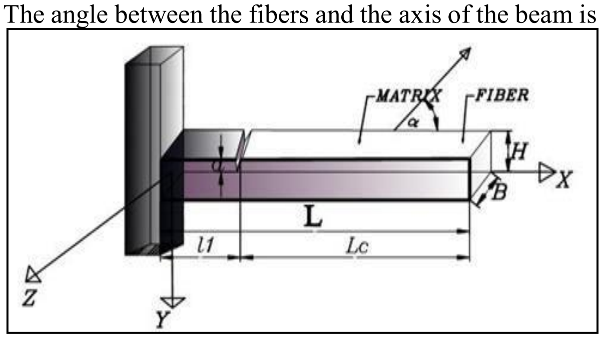

Fig. 1: Mathematical Model

C. Buckling Analysis Studies

The dynamic response of a beam for a conservative system can be formulated by means of Lagrange"s equation of motion in which the external forces are expressed in terms of time-dependent potentials

$$M\ \ q + \ K_e - P(t)\ K_g \quad q = 0$$

D. Derivation of Element Matrices

In the present analysis three nodes composite beam element with three degree of freedom (the axial displacement, transverse displacement and the independent rotation) per node is considered.

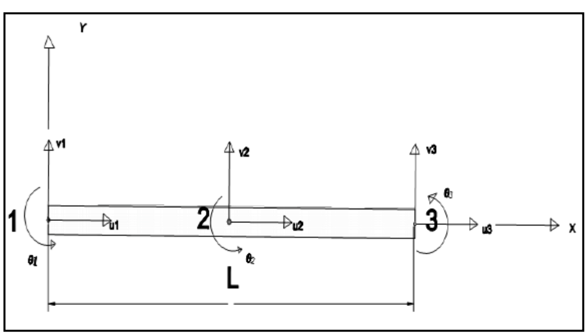

Fig. 2: Element Matrices

The linear strain can be described in terms of displacements as

$$\varepsilon = B\ \ \delta$$

E. Stress-Strain Matrix

$$D = \begin{matrix} S_{11} & S_{13} \\ S_{13} & S_{33} \end{matrix}$$

F. Element Stiffness Matrix

Element stiffness matrix for a three-nodes composite beam element with three degrees of freedom δ = (u, v, θ) at each node, for the case of bending in the x, y plan, are given

$$K_e = IS_{11}\ \ _v[B]^T D\ B\ dv$$

G. Generalized Element Mass Matrix

Element mass matrix of the non-cracked composite beam element is given:

$$M_e = \rho\ \ _v[N]^T\ N\ dv$$

Where

$$M_e = \rho\mathrm{BHL} \begin{matrix}
\frac{2}{15} & 0 & 0 & \frac{2}{15} & 0 & 0 & \frac{-1}{30} & 0 & 0 \\
0 & \frac{2}{15} & \frac{L}{180} & 0 & \frac{1}{15} & \frac{-L}{90} & 0 & \frac{-1}{30} & \frac{L}{180} \\
0 & \frac{L}{180} & \frac{L^2}{1890}+\frac{H^2}{90} & 0 & 0 & \frac{-L^2}{945}+\frac{H^2}{180} & 0 & \frac{-L}{180} & \frac{L^2}{1890}-\frac{H^2}{90} \\
\frac{2}{15} & 0 & 0 & \frac{8}{15} & 0 & 0 & \frac{1}{15} & 0 & 0 \\
0 & \frac{1}{15} & 0 & 0 & \frac{8}{15} & 0 & 0 & \frac{1}{15} & 0 \\
0 & \frac{-L}{90} & \frac{-L^2}{945}+\frac{H^2}{180} & 0 & 0 & \frac{2L^2}{1890}+\frac{2H^2}{90} & 0 & \frac{L}{90} & \frac{-L^2}{945}+\frac{H^2}{180} \\
\frac{-1}{30} & 0 & 0 & \frac{1}{15} & 0 & 0 & \frac{2}{15} & 0 & 0 \\
0 & \frac{-1}{30} & \frac{-L}{180} & 0 & \frac{1}{15} & \frac{-L}{90} & 0 & \frac{2}{15} & \frac{-L}{180} \\
0 & \frac{L}{180} & \frac{L^2}{1890}-\frac{H^2}{90} & 0 & 0 & \frac{-L^2}{945}+\frac{H^2}{180} & 0 & \frac{-L}{180} & \frac{L^2}{1890}+\frac{H^2}{90}
\end{matrix}$$

1.1 H. Geometrical Stiffness Matrix

Geometrical stiffness matrix of the composite beam element is given as

$$K_g = T\ \int_0^L N'^{\ T}\ N'\ dx$$

1) Computational procedure for a cracked composite beam

A computer program is developed to perform all the necessary computations in MATLAB environment. In the initialization phase, geometry and material parameters are specified. For example for a Euler–Bernoulli composite beam model with localized crack, material parameters like modulus of elasticity, the modulus of rigidity, the Poisson ratio and the mass density of the composite beam material and geometric parameters like dimensions of the composite beam, also the specifications of the damage like size of the crack, location of the crack and extent of crack are supplied as input data to the computer program. The beam is divided into n number of elements and n+1 number of nodes. The elements of the mass matrix, elastic stiffness matrix and geometric stiffness matrix are formulated according to above expression and are obtained the non-dimensional natural frequencies and buckling load for non-cracked and

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cracked composite beam element. The program uses the MATLAB function, "Gauss Quadrature" to carry out the integration part. Element matrices are assembled to obtain the global matrices. Boundary conditions are imposed by elimination method. For Euler–Bernoulli composite beam with fixed-free end conditions the first three rows and columns of the global matrices are eliminated to obtain the reduced matrices.

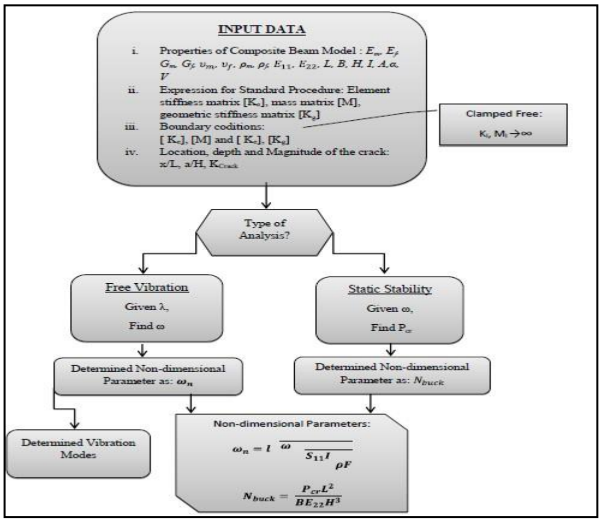

Fig. 1: Flow Chart of the Program

4. Results and Discussions

4.1 Convergence Study

The convergence study is carried out for the free vibration of cracked composite beam and omitted here for sake of brevity. Based on this study, a mesh of 12 elements shows good convergence of numerical solutions for free vibration of cracked composite beam. Convergence of non-dimensional free vibration frequencies of cracked composite beam for different angle of fibers V = 0.1, a/H = 0.2, Em = 2.756; Ef = 275.6; v =.2;
G =1.036; G =114.8; ρ =1600; ρ = 900;

Table 1: Mesh Division

Mesh Division	Non dimensional frequencies for different angle of fibers"α"(degrees)		
	α = 0	α = 15	α = 30
2 elements	1.5982	1.6703	1.7125
4 elements	1.6815	1.7255	1.7732
8 elements	1.6995	1.7257	1.7748
12 elements	1.7055	1.7245	1.7743
Krawczuk & Ostachowicz	1.7055	1.7245	1.7743

4.2 Comparison with Previous Studies

1) Vibration Analysis Studies

Here table 2 below shows the Comparison of First three Non-dimensional natural frequencies of the non-cracked composite beam and table 3 below shows the Comparison of Non-dimensional natural frequencies of the cracked composite beam as a function of the angle of fibers.

2) Buckling Analysis Studies

In this buckling analysis study, the results of non-cracked composite beam obtained with the present element are compared with the analytical results of Reddy (1997) and Ozturk & Sabuncu (2005).

Table 4: Buckling Analysis Study

Angle of fibers(degree)	Present FEM	Ozturk & Sabuncu(2005)	Reddy (1997)
0	4.9984	5.1404	5.14
30	1.6632	-	-
60	0.3891	-	-
90	0.2006	0.2056	0.205

5. Conclusion and Future work

5.1 Conclusion

The following conclusions can be made from the present investigations of the composite beam finite element having transverse non-propagating one-edge open crack. This element is versatile and can be used for static and dynamic analysis of a composite or isotropic beam.

- From the present investigations it can be concluded that the natural frequencies of vibration of a cracked composite beam is not only the functions of the crack locations and crack depths but also the functions of the angle of fibers and the volume fraction of the fibers. The presence of a transverse crack reduces the natural frequencies of the composite beam.
- The rate of decrease in the natural frequency of the cracked composite beam increases as the crack position approaches the fixed end.
- The intensity of the reduction in the frequency increases with the increase in the crack depth ratio. This reduction in natural frequency along with the mode shapes of vibrations can be used to detect the crack location and its depth.
- When, the angle of fibers (α) increases the values of the natural frequencies also increase. The most difference in frequency occurs when the angle of fiber (α) is 0 degree. This is due to the fact that the flexibility of the composite beam

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due to crack is a function of the angle between the crack and the reinforcing fibers.

- The effect of cracks is more pronounced near the fixed end than at far free end. It is concluded that the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free end, respectively.
- The decrease of the non-dimensional natural frequencies depends on the volume fraction of the fibers. The non-dimensional natural frequency is maximum when the volume fraction of fiber is approximately 45%. This is due to the fact that the flexibility of a composite beam due to crack is a function of the volume fraction of the fibers.
- Buckling load of a cracked composite beam decrease with increase of crack depth for crack at any particular location due to reduction of stiffness.
- When, angle of fibers increase the values of the buckling loads decrease. This is due to the fact that for 0 degree orientation of fibers, the buckling plane normal to the fibers is of maximum stiffness and for other orientations stiffness is less hence buckling load is less.

5.2 Scope of Future Work

- The vibration and stability results obtained using this formulation can be verified by conducting experiments.
- The dynamic stability of the composite beam with cracks
- Static and dynamic stability of reinforced concrete beam with cracks.
- The dynamic stability of beam by introducing slant cracks (inclined cracks) in place of transverse crack.

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I am pursuing master of technology in (structural design) department of civil engineering from SSSUTMS, Sehore and completed my B.Tech. from Aryabhatta knowledge university,Patna (Bihar), India. I have qualified the GATE exam two times in year 2019 and 2020.

I have completed my Ph.D. from Pacific University, Rajasthan in March 2015, Master of Technology from MANIT, Bhopal and Bachelor of Engineering from SATI, Vidisha. I have published many papers in reputed international and national journal and presented paper in conferences/workshop. I am also the member of Geotechnical Society of Engineers and Indian concrete Institute.

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Table 2: Comparison of First three Non-dimensional natural frequencies of the non-cracked composite beam as a function of the angle of fibers α, where Value of V=0.10 and 0.30.

Angle of Fibers (degrees)		Present Analysis			Krawczuk & Ostachowicz (1995)		
	Volume of Fraction V	1st Non-Dimensional Nat. freq	2nd Non-Dimensional Nat. freq	3rd Non-Dimensional Nat. freq	1st Non-dimensional Nat. freq	2nd Non-dimensional Nat. freq	3rd Non-dimensional Nat. freq
0	0.10	1.8798	4.6566	7.6681	1.85145	4.52827	7.71888
15		1.8243	4.5300	7.4841	1.81768	4.51477	7.51418
30		1.6655	4.1530	6.9033	1.65453	4.12945	6.89687
45		1.4342	3.5854	5.9833	1.38995	3.53323	5.97735
60		1.2083	3.0230	5.0513	1.15370	3.01580	5.01780
75		1.0998	2.7514	4.5973	1.08133	2.74520	4.57040
90		1.0881	2.7205	4.5410	1.08007	2.71020	4.51710
0	0.30	1.8771	4.6113	7.5073	1.85145	4.52827	7.64894
15		1.8188	4.4873	7.3447	1.81768	4.44477	7.37372
30		1.6484	4.0982	6.7804	1.65453	4.02945	6.92680
45		1.3886	3.4682	5.7818	1.38995	3.43323	5.85710
60		1.1068	2.7684	4.6260	1.15370	2.71580	4.76640
75		0.948	2.3713	3.9632	1.08133	2.27052	4.04030
90		0.9307	2.3263	3.8831	1.08007	2.21720	3.97620

Table 3: Comparison of Non-dimensional natural frequencies of the cracked composite beam as a function of the angle of fibers (α) for several values of the crack depth a/H =0.2, 0.4, 0.6 (value fraction of fibers V = 10%, crack location *x*/L = 0.1)

Angle of Fibers (degrees)	Relative Cracked Depth (a/H)	Present Analysis		Krawczuk & Ostachowicz (1995)	
		1st Non-dimensional Nat. freq	2nd Non-dimensional Nat. freq	1st Non-dimensional Nat. freq	2nd Non-dimensional Nat. freq
0	0.2	1.7070	4.5477	1.7100	4.5400
15		1.7260	4.5656	1.7260	4.5600
30		1.7755	4.6064	1.7705	4.6000
45		1.8337	4.6489	1.8297	4.6400
60		1.8738	4.6771	1.8728	4.6700
75		1.8875	4.6858	1.8805	4.6808
90		1.8886	4.6898	1.8806	4.6820
0	0.4	1.4110	4.4495	1.4150	4.4315
15		1.4428	4.6590	1.4458	4.4500
30		1.5359	4.5075	1.5452	4.5000
45		1.6723	4.5654	1.6753	4.5700
60		1.7939	4.6232	1.7940	4.6232
75		1.8432	4.6492	1.8332	4.6492
90		1.8479	4.6497	1.8409	4.6497
0	0.6	1.2216	4.2150	1.1316	4.2210
15		1.2530	4.2414	1.1930	4.2400
30		1.3484	4.3243	1.3184	4.3300
45		1.4996	4.4474	1.5004	4.4500
60		1.6511	4.5732	1.6523	4.5600
75		1.7189	4.6233	1.7189	4.6200
90		1.7256	4.6272	1.7356	4.6272